Complex networks coarse-graining by Laplacian Renormalization Group

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Complex networks usually exhibit a rich architecture organized over multiple intertwined scales. Information pathways are expected to pervade these scales reflecting structural insights that are not manifest from analyses of the network topology. Moreover, small-world effects correlate with the different network hierarchies complicating the identification of coexisting mesoscopic structures and functional cores. We present a communicability analysis of effective information pathways throughout complex networks based on information diffusion to shed further light on these issues. This will lead us to a formulation of a new and general renormalization group scheme for heterogeneous networks. The Renormalization Group (RG) is the cornerstone of the modern theory of universality and phase transitions, a powerful tool to scrutinize symmetries and organizational scales in dynamical systems. However, its network counterpart is particularly challenging due to correlations between intertwined scales. To date, the explorations are based on hidden geometries hypotheses. Here, we propose a Laplacian RG diffusion-based picture for complex networks, defining the supernodes concept à la Kadanoff, the equivalent momentum space procedure à la Wilson for graphs, and applying this RG scheme to real networks in a natural and parsimonious way (see Fig.1).

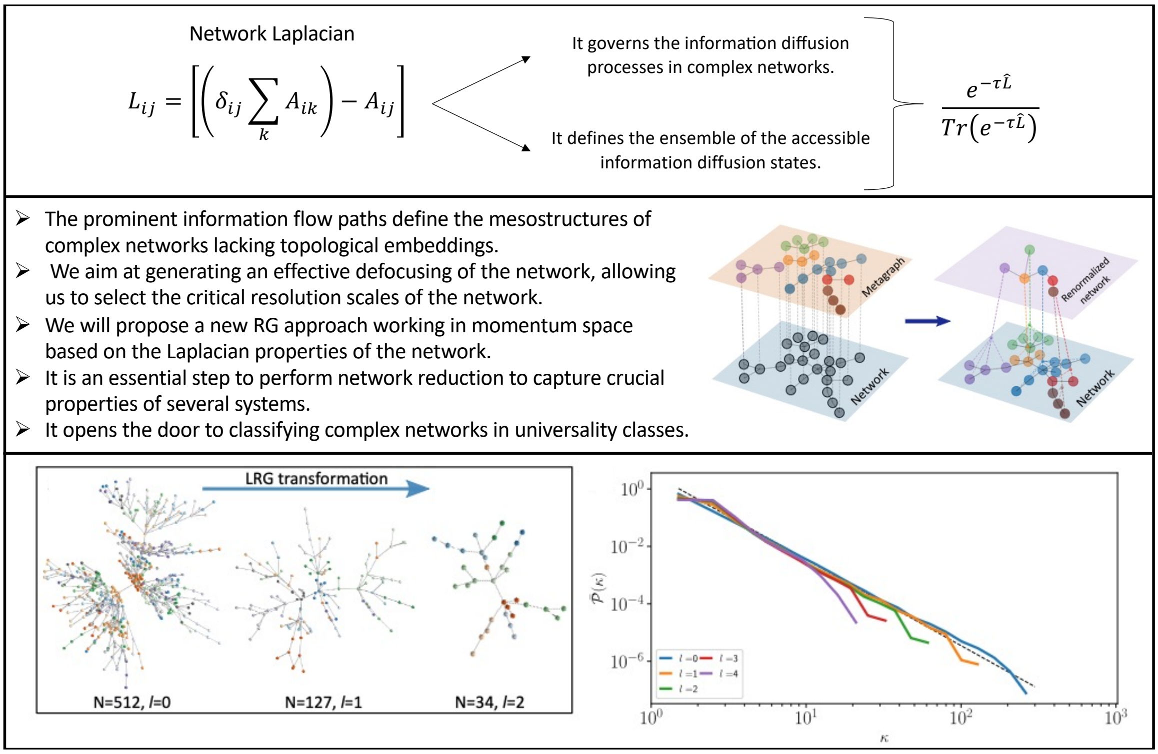

Fig.1. Coarse graining by Laplacian Renormalization Group: the case of Barabási-Albert networks.

References [1] *P. Villegas, A. Gabrielli, G. Caldarelli, T. Gili,* ***Laplacian paths in complex networks: Information core emerges from entropic transitions.*** *P. Villegas, A. Gabrielli, F. Santucci, G. Caldarelli, T. Gili, Phys. Rev. Research 4, 033196 (2022). [2] P. Villegas, T. Gili, G. Caldarelli, A. Gabrielli,* ***Laplacian Renormalization Group for heterogeneous networks****, https://doi.org/10.48550/arXiv.2203.07230 (2022).*